



# Mathematics Education and Graph Theory

Proceedings of International Seminar  
on Mathematics Education and Graph Theory  
June 9, 2014

Editors:  
Mustangin  
Abdul Halim Fathani

# **Mathematics Education and Graph Theory**

PROCEEDINGS OF INTERNATIONAL SEMINAR  
ON MATHEMATICS EDUCATION AND GRAPH THEORY



**Unit of Publication  
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Faculty of Teacher Training and Education  
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## **MATHEMATICS EDUCATION AND GRAPH THEORY**

### **Proceedings of International Seminar on Mathematics Education and Graph Theory**

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Islamic University of Malang, 2014

*These proceedings contain the full texts of paper and talks presented  
in the International Seminar on Mathematics Education and Graph Theory  
on June 9, 2014*

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## PREFACE

These proceedings contain the full text of papers and talks presented in the International Seminar on Mathematics Education and Graph Theory. This seminar was held in conjunction with the International Workshop on Graph Masters. The workshop was held on June 7–8, 2014, while the seminar was on June 9, 2014. These events were organized by Islamic University of Malang (Unisma) in cooperation with Indonesian Combinatorial Society (InaCombS).

The workshop and the seminar would not have been possible without the time and energy put forth by the invited speakers. The invited speakers of the workshop were: **Mirka Miller**, University of Newcastle, Australia; **Joseph Miret**, Universitat de Lleida, Spain; **Christian Mauduit**, Institut de Mathematiques de Luminy, France; **Edy T. Baskoro**, Bandung Institute of Technology, Indonesia; **Surahmat Supangken**, Islamic University of Malang, Indonesia; **Tri Atmojo**, State University of Semarang, Indonesia; and **Purwanto**, State University of Malang, Indonesia.

The invited speakers of the seminar were: **Juddy Anne Osborn**, University of Newcastle, Australia and **Abdur Rahman As'ari**, State University of Malang, Indonesia. The seminar was held on the area of mathematics education and graph theory. The main themes of the mathematics education seminar include topics within the following areas (but not limited to): philosophy of mathematics education, curriculum development, learning methods and strategies, learning media, development of teaching material, and assessment and evaluation of learning. The main themes covered in graph theory seminar include topics within the following areas (but not limited to): degree (diameter) problems, ramsey numbers, cycles in graphs, graph labeling, dimensions of graphs, graph coloring, algorithmic graph theory, and applications of graph theory in various fields.

We would like to thank you to the invited speakers and all presenters who have submitted papers, for their valuable and inspiring presentation. A special appreciation goes to: **Surahmat Supangken**, Rector of Unisma and **Kiki Ariyanti Sugeng**, the President of InaCombS, who have made a lot of efforts to prepare this seminar.

We also do not forget to express our gratitude to Islamic University of Malang (Unisma) for providing financial support, and to the Indonesian Combinatorial Society (InaCombS) for the support. We hope that you had a great time and valuable experience during the seminar in Malang.

Malang, July 22, 2014

**Editors**



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# Mathematics Education

PROCEEDINGS OF INTERNATIONAL SEMINAR  
ON MATHEMATICS EDUCATION AND GRAPH THEORY

***Theoretical (Conceptual) Articles***

## **PATTERN AND STRUCTURE MATHEMATICS AWARENESS CONTRIBUTED TO NUMBER SENSE EARLY CHILDHOOD**

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### **Abstract**

The main objective of the preschool program is to prepare preschool age children for early learning success at primary school level. Research has shown that this goal can be achieved if only the school has high -quality preschool programs. High-quality programs facilitate children's learning of mathematics. One of the high-quality program is involving early intervention in mathematics. Math teacher intervenes in accordance with the cognitive development of children. Early intervention in early childhood mathematics aims to observe the development of children's mathematical patterns and structures. This early intervention can overcome learning difficulties at the time of formal schooling. Awareness of mathematical patterns and structure is very important for the development of mathematical concepts, skills and proficiency. The purpose of this article is to investigate how can awareness in patterns and structures owned by early childhood contributes to the number sense.

**Keywords:** *Pattern, Structure Mathematics, Number Sense, Early Childhood, Five-Frames*

### **INTRODUCTION**

The main objective of the program is to prepare preschool age children for early learning success at primary school level. Research has shown that this goal can be achieved only if high-quality preschool programs (Barnett & Boocock, 1998). Teachers who understand child development in all fields. Teachers provide children with meaningful experiences to build on the knowledge already possessed. These include teachers who provide high-quality learning.

Research shows that high-quality programs facilitate children's learning in mathematics (Clements & Sarama, 2004). One of the high-quality program with early intervention in mathematics, such as preschool programs that integrate math practice in accordance with the development of the child can cope with learning difficulties at the time of formal schooling (Fuson, Smith, & LoCicero, 1997). Teachers know the children know about math early, when it began to think. This can be seen in two ways, observation (observing the development of motor and cognitive representations through children) and

intervention. Teachers observe the children search for mathematical thinking beyond counting, identifying shapes, and simple division, estimation, patterns, symmetry, and understanding of spatial relationships (Kyoung-Hye Seo, 2003).

Early intervention strategies to prevent early childhood mathematics learning difficulties, remediate and educators to encourage children to give children the opportunity developmental math enrichment before formal schooling (Doig, B, et al 2003). Without active intervention seems to be the kids with little mathematical knowledge at the beginning of formal schooling will remain low achievers throughout the elementary school years (Aubrey, C, 2006). The results of a brief intervention showed recurring patterns can act as an effective bridge to introduce the concept of comparison. They also show that certain representations and actions petrified student teachers to identify comparison, achieving equality between certain comparisons, and began to represent these ideas in abstract notation system (Warren, E., & Cooper, TJ 2007)

Ginsburg et al (2003) extensively studied the children of China and the U.S. for 4-5 years and how they use and understand mathematical reference during free play. The researchers found there were six main categories used in the children's understanding of mathematics, namely: (1) classification, (2) the size, (3) enumeration, (4) the pattern and shape, (5) spatial relationships, and (6) the concept of partial / entirely

The new math program children ages 4 to 8 years emphasizes structural aspects of patterns and relations, including equality, growth patterns, and functional thinking (Warren & Cooper, 2005, 2008). Awareness on the mathematical structure is very important in early childhood mathematics for competence (Mulligan, JT, & Mitchelmore, MC 2009).

## **LITERATURE REVIEW**

### **Defining Mathematical Pattern and Structure**

A mathematical pattern may be described as any predictable regularity, usually involving numerical, spatial or logical relationships (see Mulligan & Mitchelmore, 2009). In early childhood, the child has a pattern including repeating patterns (for example, ABABAB ...), the pattern of spatial structures (eg, geometric shapes), growth pattern (eg, 2, 4, 6, 8, ...), Unit size or transformation. Structure refers to the way in which the various elements are organised and related including spatial structuring (see Mulligan et al., 2003). Structural development can emerge from, or underlie mathematical concepts, procedures and relationships and is based on the integration of complex elements of pattern and structure that lead to the formation of simple generalis example, recognising structural features of equivalence,  $4 + 3 = 3 + 4$  may reflect he child's perceived symmetrical structure (see Mulligan & Mitchelmore, 2009)

### **Early Childhood Research Mathematical Pattern and Structure**

Research on early mathematics learning has often been restricted to an analysis of children's developmental levels of single concepts such as counting, but has not provided insight into common underlying processes that develop mathematical generalization (Mulligan & Vergnaud, 2006). Mathematics Education and Neurosciences (MENS) Project provide frameworks to promote 'big ideas' in early mathematics and science education (van Nes & de Lange, 2007). Algebraic thinking is thought to develop from the ability to see and represent patterns and relationships such as equivalence and functional thinking from the early childhood years (Papic, Mulligan, & Mitchelmore, 2009; Warren & Cooper, 2008).

Research in number. have all shown how progress in students' mathematical understanding depends on a grasp of underlying structure. Significant concentrations of new research with young children focused on data modeling and statistical reasoning also provide an integrated approach to studying structural development (Lehrer, 2007).

### **Structural Developmental Stages of Early Childhood**

Battista et al. (1998) found the spatial structuring abilities of students to provide the input and organization for the numerical procedures that the students used to count an array of cubes. Spatial structuring is also influenced by the students' attempts at counting, and it is based on how students physically and mentally act on a spatial configuration. Spatial structuring as a type of organization is considered to contribute to insight into important mathematical concepts such as patterning, algebra, and the recognition of basic shapes and figures (Mulligan et al., 2006). From this, Battista and Clements concluded that spatial structuring fundamentals later algebraic and spatial thinking. Several studies in this area have shown how low-achieving children tend to

not use any form of spatial or mathematical structure and instead continue to rely on

superficial features (Mulligan et al., 2005).

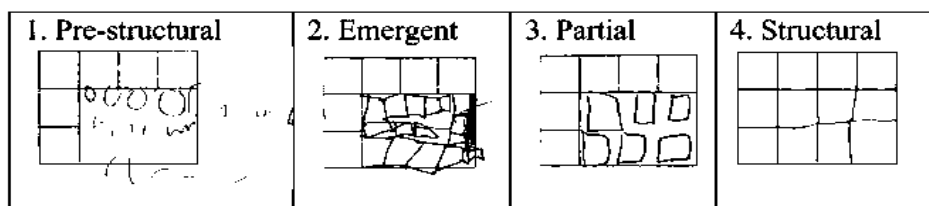


Figure 1. Typical responses by four different 6-year-old students to an area task, by stages of structural development.

Over the past decade a suite of studies with four- to nine-year olds has examined how children develop an Awareness of Mathematical Pattern and Structure (AMPS), found to be common across mathematical concepts (Mulligan, 2011; Mulligan, English, Mitchelmore). An assessment interview, the Pattern and Structure Assessment (PASA) and a Pattern and Structure Mathematics Awareness Program (PASMAT) focuses on the development of structural relationships between concepts. Tracking, describing and classifying children's models, representations, and explanations of their mathematical ideas and analysing the *structural* features of this development—are fundamentally important

Students' drawn responses and their explanations, at the three assessment points, were categorised using the levels of analysis from previous studies (Mulligan & Mitchelmore, 2009) as follows and Figure 1:

- *Pre-structural*: representations lack evidence of numerical or spatial structure
- *Emergent (inventive-semiotic)*: representations show some relevant elements but numerical or spatial structure is not represented
- *Partial structural*: representations show most relevant aspects but are incomplete
- *Structural*: representations correctly integrate numerical and spatial structural Features

### Intervention Programs

In comparison to the area of literacy, and in particular reading, there are few

comprehensive programs to assist teachers of children who experience mathematics learning difficulties. However, several effective programs have been developed such as Mathematics Recovery (Wright, 2003).

The Intervention was designed on the basis of children's existing patterning knowledge to: provide explicit opportunities to explore and develop patterning skills through problem-based task develop children's mathematical reasoning in order to provide a foundation for later mathematical learning particularly in early algebraic thinking.

Intervention group children demonstrated effective use of a unit of repeat and spatial relationships and most were also able to extend and explain growing patterns over a year later. Papic and Mulligan (2007) studied patterning strategies during the year prior to formal schooling in 53 children from two similar pre-schools. Intervention group children demonstrated effective use of a unit of repeat and spatial relationships and most were also able to extend and explain growing patterns over a year later

An important outcome of the intervention program was the development of the concept of unit of repeat (Papic, Mulligan, & Mitchelmore, 2011). When children construct a simple repetition, the pattern structure reflects equal-sized groups. This approach allows for the development of counting techniques that are more effective than unitary counting. During the intervention, children were frequently observed

### Defining Number Sense

Specific definition of number sense varies every researcher mathematics. Number sense can broadly be defined as the ease and flexibility with which children operate with numbers (Gersten & Chard, 1999). Number sense involves abilities related to counting, number patterns, magnitude, decompositions, estimating, and number transformation (Bench, 2005), Kemampuan awal kuantitatif termasuk kemampuan anak untuk substitusi dan membandingkan jumlah dengan membuat korespondensi (Clements & Sarama, 2007) Jordan et al. (2006) cite five key elements thought to govern the basic development of number sense in kindergarten children (1) correspondence principle, stable-order principle, and understanding cardinality; (2) coordinating quantities, and making numerical magnitude comparisons; (3) addition and subtraction, (4) Estimation approximating or estimating set sizes, (5) Number Patterns—copying, extending.

### Number Sense to Five-Frame Description

Five-frames will serve as a useful tool to support pre-K students' development of number sense concepts for several reasons. First, five-frames provide a basic and consistent organizational structure that inherently anchors mathematics around the number five, an important benchmark number in children's mathematical development (Novakowski 2007). Second, due to the limited number of squares in a five-frame, students are automatically constrained to working with smaller set sizes (five or fewer) that are well within their developmental counting range (Baroody 2009). Third, five-frames present opportunities for children to establish connections between different numerical representations, a critical skill in one's mathematical development (NRC 2009). Fourth, five-frames allow children to explore combinations of numbers and observe part-to-whole relationships, an important consideration during early mathematics (Hunting 2003). Finally, five-frames are visually and conceptually similar to ten-frame representations, and therefore

early exposure to five-frames will help to familiarize students with an instructional tool commonly used in later elementary mathematics.

A five-frame (see Fig. 2) is simply a 1 x 5 row of squares that allows users to place physical manipulatives (dots, counters, coins, etc.), each within a single box, to create a visual representation for numbers zero–five. Similarly, traditional ten-frames (Fig. 3) are 2 x 5 arrays are designed to serve the same purpose as five-frames, but extend representations of numbers to zero through ten.

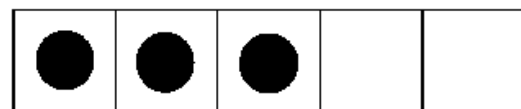


Fig. 3 Five-frame representation of the number three

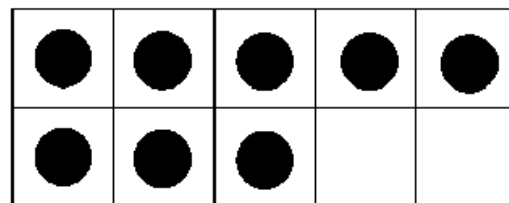


Fig. 2 Ten-frame representation of the number eight

Five-frames, describing their potential as both instructional scaffolds (e.g., assisting children in counting, partitioning, and tagging concrete objects) as well their capacity to serve as concrete analogs (Boulton-Lewis 1998) to represent numerical quantities and help students establish connections between different numerical representations

### Awareness of Pattern and Structure in Early Mathematical and Number Sense.

We define the way a mathematical pattern is organised as its *structure*. Mathematical structure is most often expressed in the form of a generalisation numerical, spatial or logical relationship which is always true in a certain domain.

An example of pattern and structure in early mathematics learning, consider the



rectangle shown in Figure 4. The pattern of  $3 \times 5$  squares is obvious to adults, but not to young students (Outhred & Mitchelmore, 2000). They apparently do not perceive its implicit structure: three rows of five equally sized squares (or five columns of three) with their sides aligned vertically and horizontally. Repetition (of individual rows or columns) and spatial relationships (congruence, parallels and perpendiculars) are the essential structural features here.

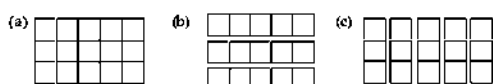


Figure 4. Rectangular grid perceived as (a)  $3 \times 5$ , (b) 3 rows of 5, (c) 5 columns of 3.

Figures 5 to 8 show typical examples of developmental features of students' (Awareness of Mathematical Pattern and Structure) AMPS in response to Item *Visual Memory Grid* ( $2 \times 3$  in PASA1 and  $3 \times 4$  in PASA2) Provide Student Recording Sheet, pencil and eraser.. In Figure 5 the student guesses the number of squares as "15" and draws single unit squares in a row (with some replication of shape) without 2-dimensional structure. Interestingly Figure 3 presents the groupings of 3 and 4 units of the grid as a border. Figure 7 shows the structure of the grid but additional units are provided, again showing "crowding". Figure 8 presents accurate alignment of a  $3 \times 4$  array as the student explains the representation as "3 by 4" rows sequentially drawn (Mulligan, et al 2011)

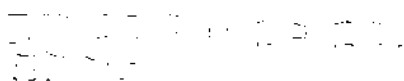


Figure 5. Pre-structural



Figure 6. Emergent

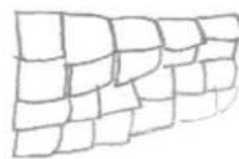


Figure 7. Partial

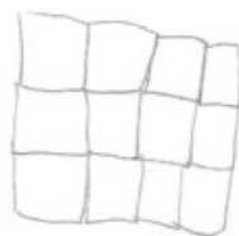


Figure 8. Structural

Observations made in kindergarten Permata Iman 3 Malang showed in Figure 9: Students named G drew a pattern grow box representation of elements (boxes) and a pattern of dots in a triangle. Structure of the existing rudimentary numerical growth pattern which is seen in this figure is correct, while the lateral growth pattern has not been true, this means the numerical representation of under-represented. In Figure 9 student called G, on Pre-structural stage, is not still using the time limit for drawing a box next to an existing image. Students drew a pattern of growth based on the results of the image that students have entered the stage of Emergent. In the second image grow box and circle pattern shows some (elements of) the relevant boxes with the structure given in the form of squares and a pattern of dots in a triangle.

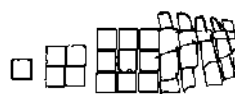


Figure 9: Pre-structural stage students draw the pattern grows

Figure 10: Students named D drew representation grow pattern elements (boxes) and a pattern of dots in a triangle. Structure of the existing rudimentary numerical growth pattern which is seen in

this figure is correct, while the lateral growth pattern has not been true, this means the numerical representation is under-represented. In Figure 10 students named D is not at the stage of pre-structural who still had to limit use when drawing the box next to images that have been available. Students drew a pattern of growth based on the results of the image that showed students have entered the stage of Emergent. In the second image grow box and circle pattern shows some (elements of) the relevant boxes with the structure given in the form of squares and circles (dots). In the second picture the pattern of squares and circles indicate growth is still at the stage of some partial (elements) into the box is still not like the pictures that have been given.

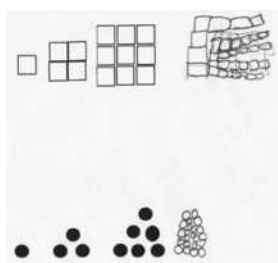


Figure 10: Emergent stage students draw the pattern grows

Figure 11: Students named C drew a pattern grow box representation of elements (boxes) has structural developmental stages (S). True representation integrating numerical and spatial structure element. Spatial numerical structure and the pattern are correct since it grows upward and laterally correct box size is appropriate, but the pattern of dots in the triangular structure of numerical and spatial is only closer to the truth. It is still not the such image that has been given.



Figure 11: Structural stage students draw the pattern grows

## DISCUSSION

Students' mathematical thinking in early childhood can be described in terms of growing awareness of pattern and structure. The paper hopes to provide inspiration to build awareness of pattern and structure in early childhood. Five-frame, describe their potential as both an instructional scaffolds (e.g., helping children in counting, partitioning, and labeling concrete objects) as well as their ability to serve as a replacement for concrete (Boulton-Lewis 1998). Five-Frame helps young children build relationships between numerical representations, concrete objects and verbal counting. Further research needs to (a) which one should be given first Five-Frame or early child unitizing has started to draw some squares to cover this shape? (b) When (how old) had early childhood Awareness of Mathematical Pattern and Structure (AMPS) based on the level of structural development? (c) How has the follow-up of early childhood with a structural level already owned?

## CONCLUSION

Before outlining the overall conclusion of this paper, it should be noted that this is just a conjecture that is supported by the observation of a kindergarten school. The pattern and structure mathematics awareness on task areas: Unitizing someone has started to draw some squares to cover this shape. Finish drawing the squares. Students' responses and their explanations drawn, at the three assessment points, were categorized using the levels of analysis from previous studies which follows Figure 1: Pre-structural, Emergent, Partial-structural, and Structural (Mulligan & Mitchelmore, 2009). (a) Each level is the level of awareness of children's mathematical patterns and structures that affect the child's ability in spatial and numerical representations. At this stage of structural development of early childhood can integrate elements of numerical and spatial structure. (b) Awareness of Mathematical Pattern and Structure (AMPS) coordinating early contributes quantities, numerical

magnitude making comparisons; addition and subtraction, estimation of approximating or sets estimating sizes, number patterns-copying, extending. Awareness of Mathematical Pattern and Structure (AMPS) in children early childhood contributes to the number sense.

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